A Study of Divide and Distribute Fixed Weights and its Variants

Cayden R. Codel and Marijn J. H. Heule

Based on the PoS submission found at http://crcodel.com/research/ddfw_pos.pdf and the larger research thesis found at http://crcodel.com/research/ddfw_thesis.pdf.

Introduction

We studied the Divide and Distribute Fixed Weights (DDFW) stochastic local search algorithm

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In local minima, DDFW distributes weight from satisfied to unsatisfied clauses

We studied how to best flip variables and distribute weight by testing DDFW against modern hard benchmarks

The DDFW algorithm

Variable flip variants

Weight redistribution variants

The DDFW algorithm

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DDFW flips variables which most reduce the unsatsfied clause weight (W_U)

In local minima, DDFW moves weight from one satisfied neighbor to each unsatisfied clause

DDFW is the only SLS algorithm in the UBCSAT framework to efficiently solve the $n = 7824$ Pythagorean triples instance

The DDFW algorithm as pseudocode

Algorithm 1: DDFW

Input: CNF formula $\mathcal{F}$
Set all clause weights to $w_{\text{init}} = 8$
$\alpha \leftarrow$ randomly generated truth assignment
for MAX-FLIPS *times* **do**
  if *assignment* α *satisfies* $\mathcal{F}$ **then** return α
  if *flipping a variable reduces* W_U **then**
    Flip a literal that reduces W_U the most
  else
    for *each unsatisfied clause* C_j **do**
      $C_k \leftarrow$ maximum-weight neighbor of C_j
      if *weight of* $C_k > w_{\text{init}}$ **then**
        Transfer a weight of 2 from C_k to C_j
      else
        Transfer a weight of 1 from C_k to C_j
return "No satisfying assignment"

The DDFW algorithm as pseudocode

Algorithm 2: DDFW

Input: CNF formula $\mathcal{F}$
Set all clause weights to $w_{\text{init}} = 8$
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Benchmark set

Ten encodings of matrix multiplication challenges

Ten random 3-SAT instances from the 2018 SAT Competition

Two encodings of the $n = 7824$ Pythagorean triples problem

Two encodings of asias and three of Steiner triple problems

All CNFs can be found at
`https://github.com/marijnheule/benchmarks` and
`http://satcompetition.org`

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Idea: flip W_U-reducing variables probabilistically?

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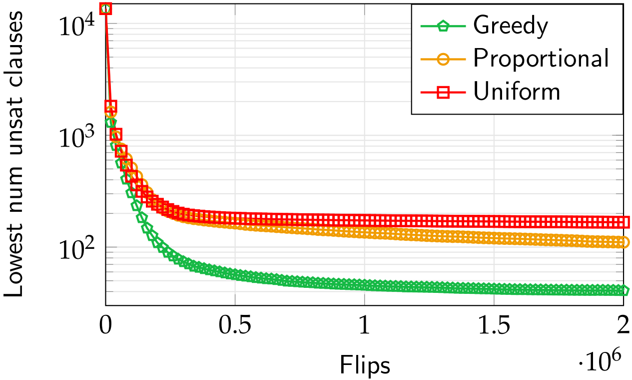
Idea: flip W_U-reducing variables probabilistically?

We investigated a uniform and a weighted probability distribution

Variable flip variant experimental results



Averaged over all problem instances. The greedy (original) method performed significantly better than the variants.

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DDFW distributes 1 or 2 units of weight between clauses

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We can generalize to a linear rule:

Algorithm 4: Linear weight transfer rule

```
if weight of unsat neighor C_k > w_init then
    Transfer a weight of a_> × w(C_k) + c_> from C_k to C_j
else
    Transfer a weight of a_≤ × w(C_k) + c_≤ from C_k to C_j
```

Weight redistribution variants experimental results

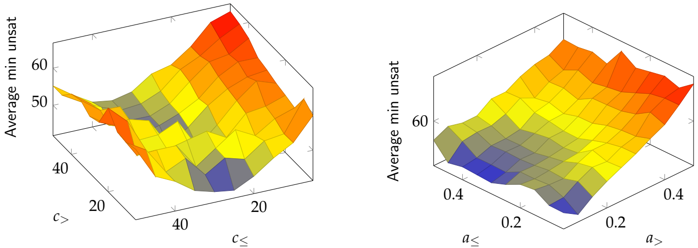
$w_{\text{init}} = 100$. Results averaged over all instances and 100 runs per instance, five million flip timeout

Distribution policy	Avg lowest unsat	Solve %
Original DDFW	36.57	0.85
$(c_{>}, c_{\leq}) = (10, 25)$	29.16	1.7
$(a_{>}, a_{\leq}) = (0.05, 0.05)$	23.82	2.96
$(a_{>}, c_{>}) = (a_{\leq}, c_{\leq}) = (0.1, 5)$	22.05	2.67

Linear rule performed about 40% better

Weight redistribution variants experimental results

Parameter searches across the matrix multiplication challenges



Note greater effect of $a_>$ on shape of right plot, while $c_\leq$ determines shape of left plot

Weight redistribution variants continued

Can also distribute weight from entire neighborhoods

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Experimental results disappointing but show some promise

Distribution policy	Avg lowest unsat	Matrix lowest unsat
Original DDFW	36.57	57.02
Proportional	46.91	27.0
Direct	45.29	33.91

Conclusions and future work

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Spreading weight across more clauses in a neighborhood could cause DDFW to escape local minima faster